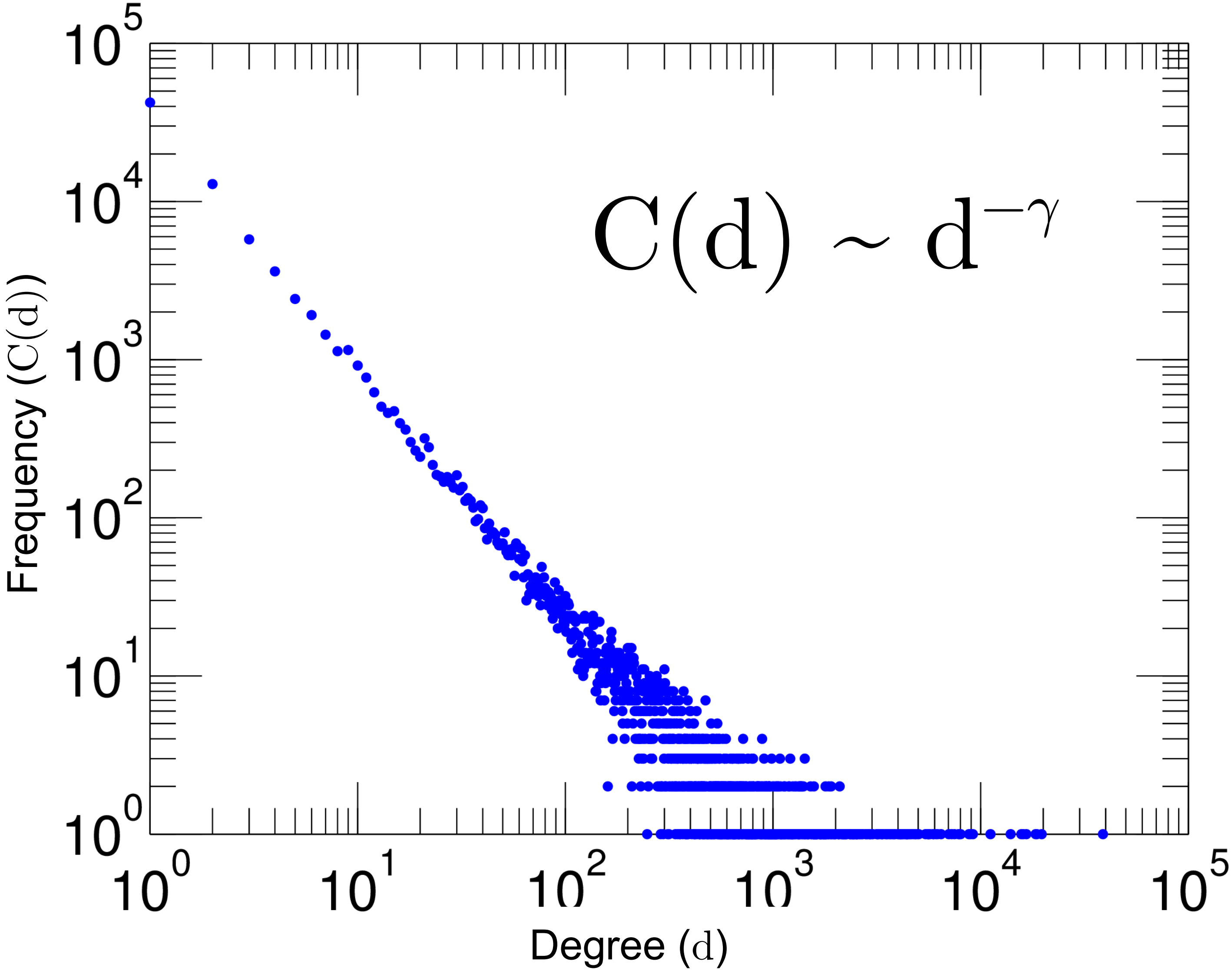
Preferential Attachment in Online Networks: Measurement and Explanations

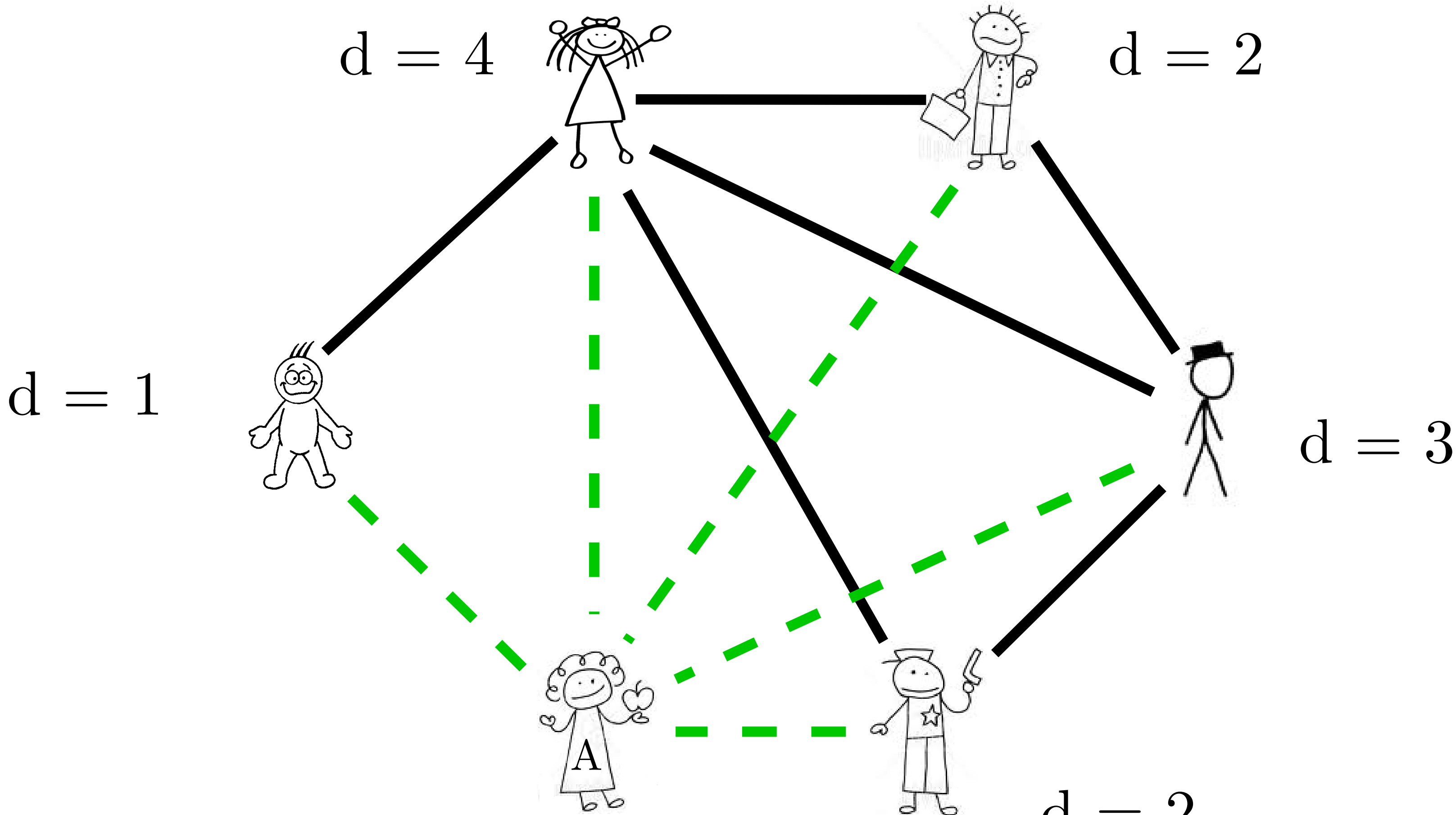
Based on:
Preferential Attachment in Online Networks: Measurement and Explanations. Jérôome Kunegis, Marcel Blattner, Christine Moser, In: Proc. Web Sci. Conf., pp. 205–214, 2013.
[http://dl.acm.org/citation.cfm?id=2464514]
[https://arxiv.org/abs/1303.6271]

Power Laws – Scale Free Networks



http://konect.cc/plots/degree/

Preferential Attachment Model



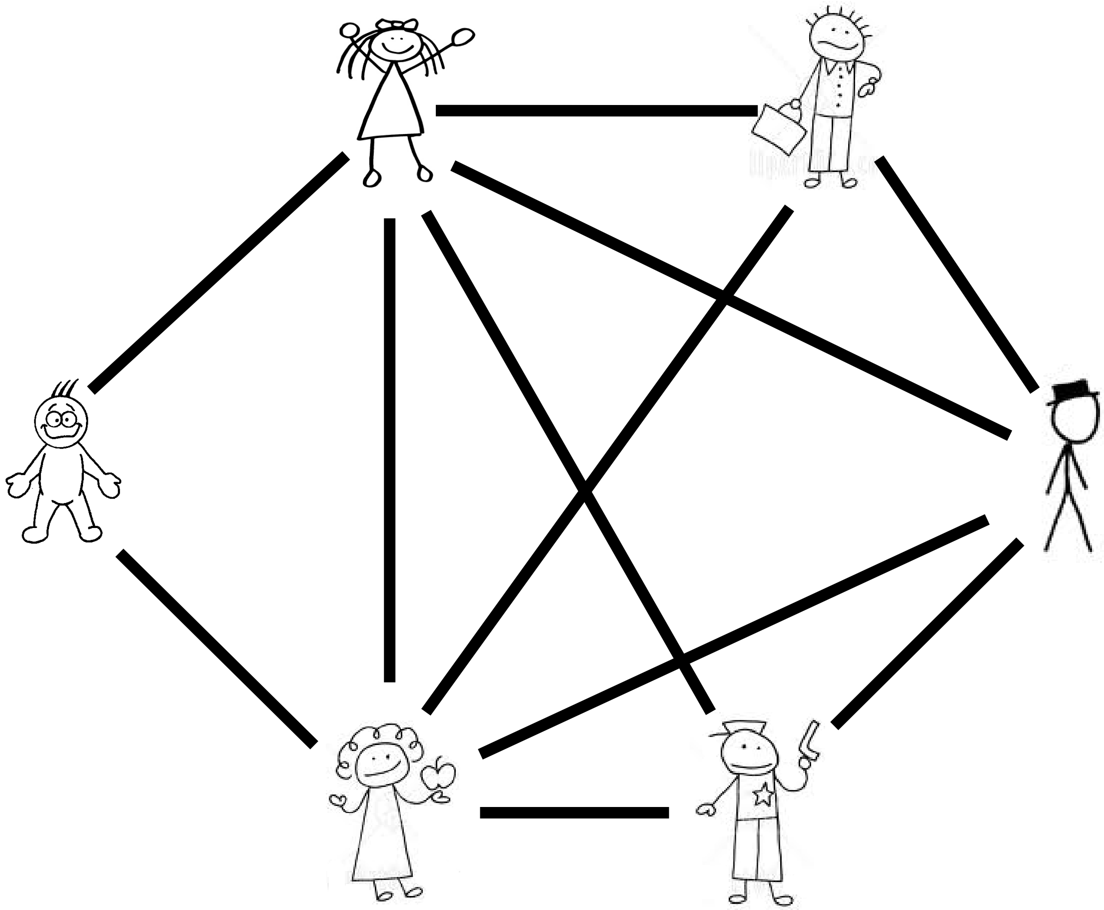
$$P(\{A,\ i\}) \sim d(i)$$

Linear vs Nonlinear Preferential Attachment

$f(\mathrm{d}) \sim 1$	Erdős–Rényi model [1]
$f(\mathrm{d}) \sim \mathrm{d}^{\beta},\ 0 < \beta < 1$	Sublinear model [2]
$f(\mathrm{d}) \sim \mathrm{d}$	Barabási–Albert model [3]
$f(\mathrm{d}) \sim \mathrm{d}^{\beta},\ \beta > 1$	Superlinear model [4]

[1] On Random Graphs I. Paul Erdős & Alfréd Rényi, *Publ. Math Debrecen* 6 (1959), 290–197.
[2] Random Networks with Sublinear Preferential Attachment: Degree Evolutions. *Electrical J. of Probability* 14 (2009), 1222–1267.
[3] Emergence of Scaling on Random Networks. Albert-László Barabási & Réka Albert, *Science* 286, 5439 (1999), 509–512.
[4] Random Trees and General Branching Processes. *Random Struct. Algorithms* 31, 2 (2007), 186–202.

Erdős–Rényi Model (1959)



$$P(\{i, j\}) = p$$

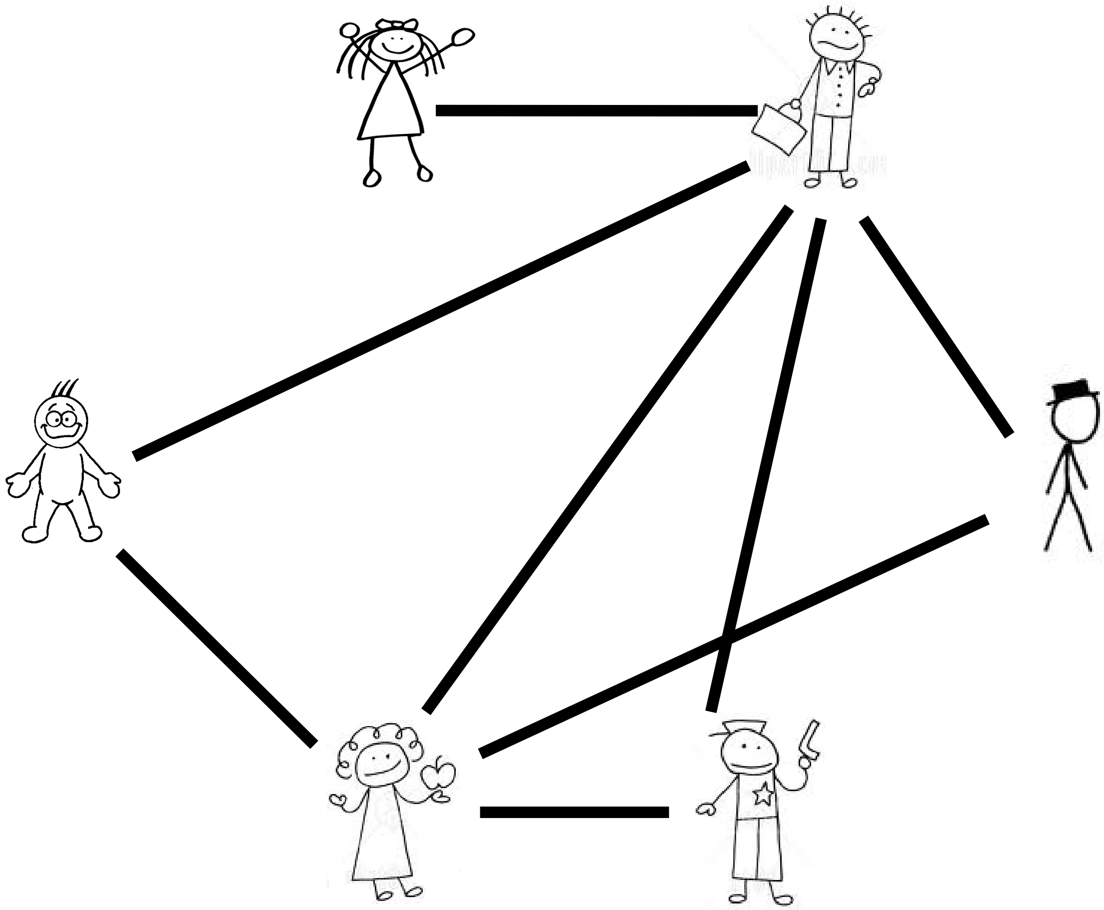
- Every edge equiprobable
- No structure

- Binomial degree distribution

$$C(d) \sim p^{d} (1 - p)^{|V| - 1 - d}$$

[1] On Random Graphs I. Paul Erdős & Alfréd Rényi, *Publ. Math Debrecen* 6 (1959), 290–197.

Barabási–Albert Model (1999)



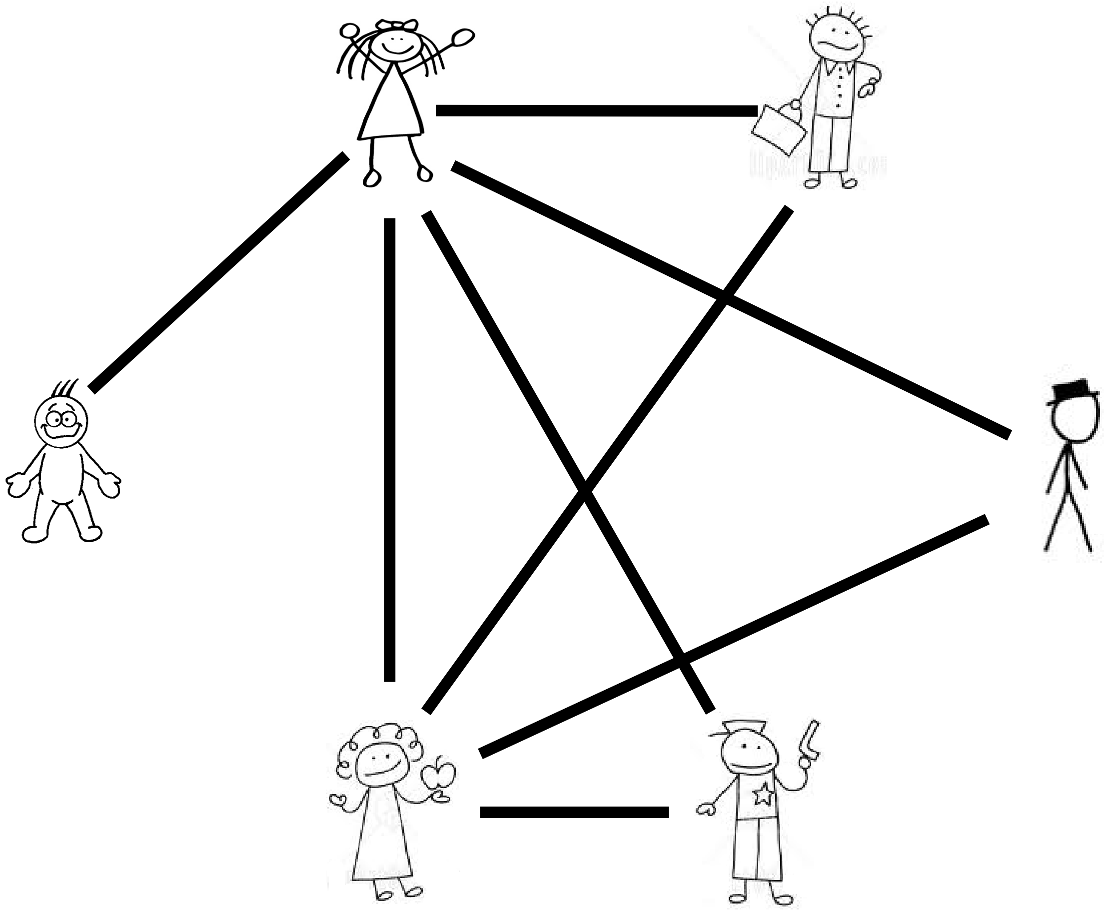
[1] Emergence of Scaling on Random Networks. Albert-László Barabási & Réka Albert, *Science* 286, 5439 (1999), 509–512.

$$P(\{A, i\}) \sim d(i)$$

- Generative model
- Scale-free network
- Power law degree distribution

$$C(d) \sim d^{-\gamma}$$

Sublinear Model



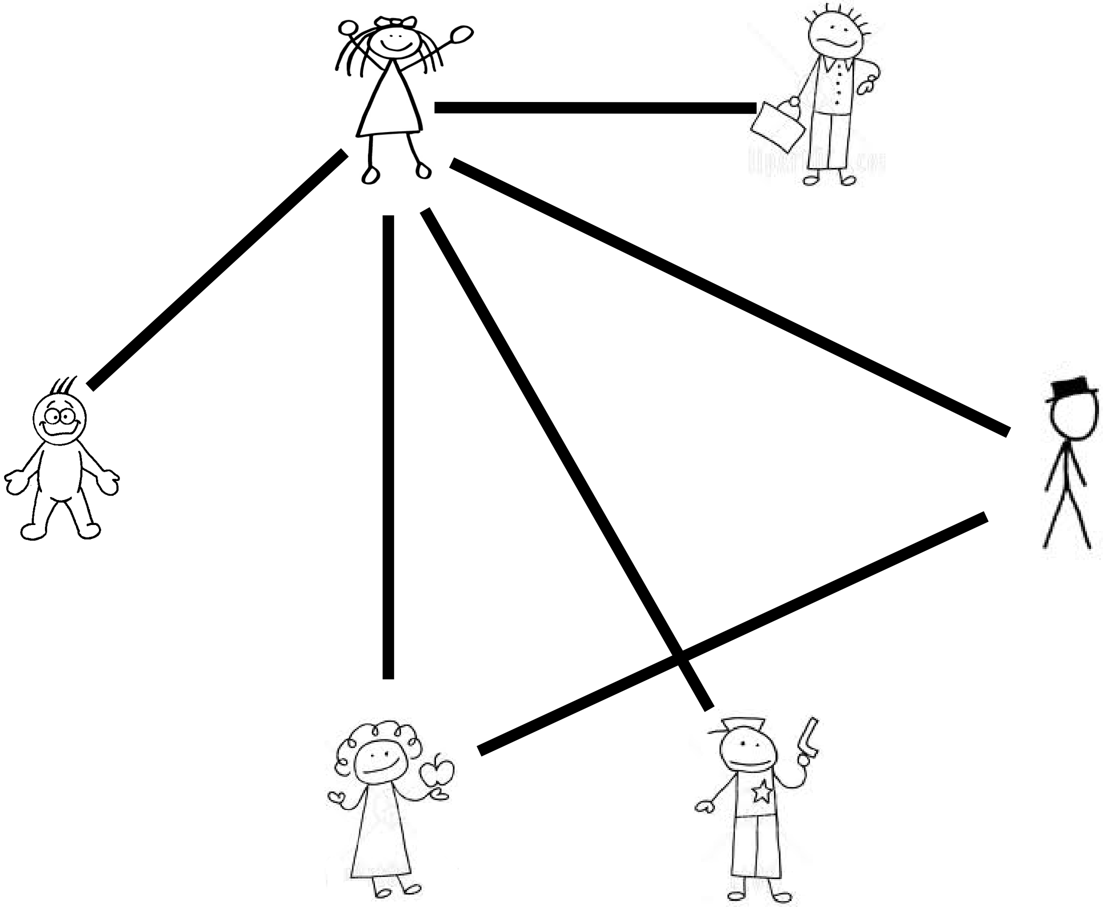
$$P(\{A, i\}) \sim d(i)^{\beta}$$
$$0 < \beta < 1$$

- Stretched exponential degree distribution [1, Eq. 94]

[1] Evolution of Networks. *Adv. Phys.* 51 (2002), 1079–1187.

[2] Random Networks with Sublinear Preferential Attachment: Degree Evolutions. *Electrical J. of Probability* 14 (2009), 1222–1267.

Superlinear Model

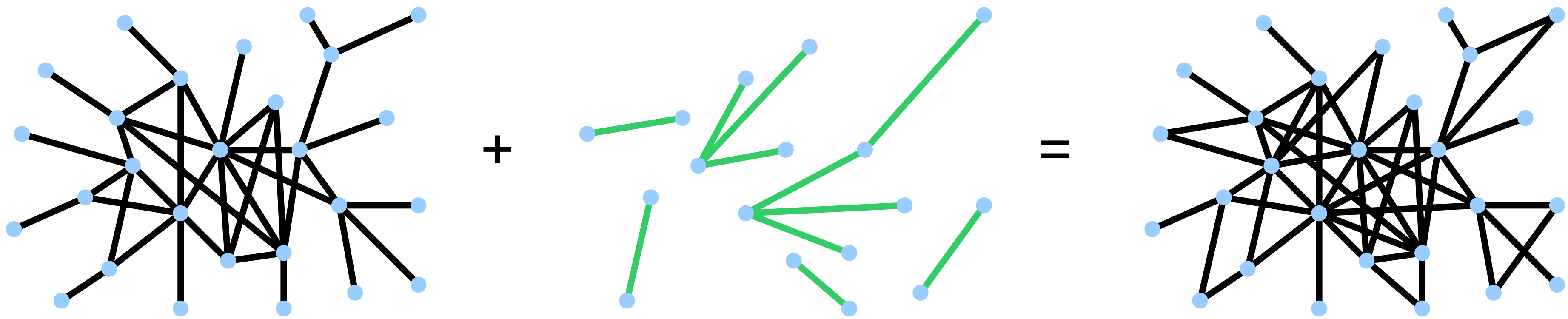


$P(\{A, i\}) \sim d(i)^{\beta}$
$\beta > 1$

- A single node attracts 100% of edges asymptotically
- Power law degree distribution in the pre-asymptotic regime

[1] Random Trees and General Branching Processes. *Random Struct. Algorithms* 31, 2 (2007), 186–202.

Temporal Network Data



Network at time t_1
Degrees $\mathrm{d}_1(u)$

Added edges
Degrees $\mathrm{d}_2(u)$

Network at time t_2
Degrees $\mathrm{d}_1(u) + \mathrm{d}_2(u)$

$$\text{Hypothesis: } \mathrm{d}_2 = \alpha \, \mathrm{d}_1{}^{\beta}$$

Empirical Measurement of *β*

$$\mathrm{d}_2 = e^{\alpha}\,(1 + \mathrm{d}_1)^{\beta} - \lambda$$

Find (α, β) using least squares:

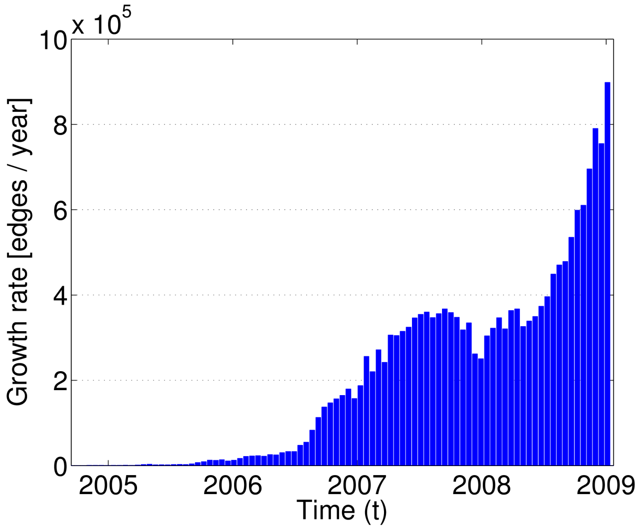
$$\min_{\alpha,\,\beta} \sum_{u\,\in V} (\alpha + \beta\,\ln[1 + \mathrm{d}_1(u)] - \ln[\lambda + \mathrm{d}_2(u)])^2$$

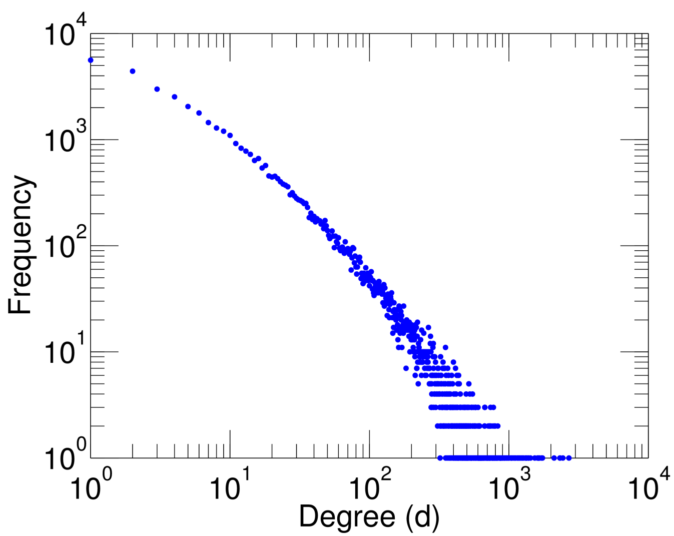
$$\varepsilon = \exp\left\{\sqrt{1\,/\,|\mathrm{V}| \sum_{u\,\in V} (\alpha + \beta\,\ln[1 + \mathrm{d}_1(u)] - \ln[\lambda + \mathrm{d}_2(u)])^2}\right\}$$

Example Network: Facebook Wall Posts

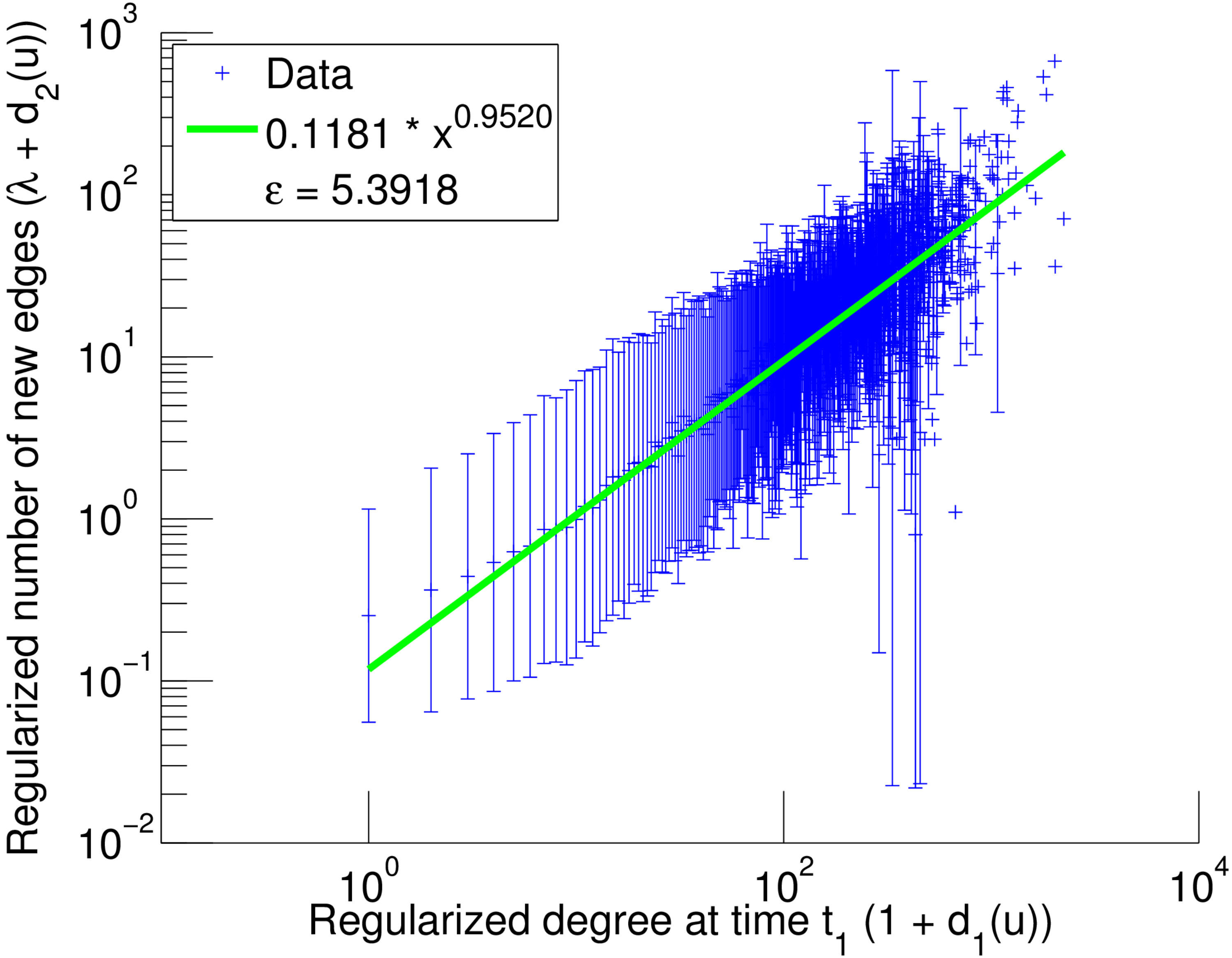
Description:	User–user wall posts
Format:	Edges are directed
Edge weights:	Multiple edges are possible
Metadata:	Edges have timestamps
Size:	63,891 vertices
Volume:	876,993 edges
Average degree:	27.45 edges / vertex
Maximum degree:	2,696 edges

http://konect.cc/networks/facebook-wosn-wall/

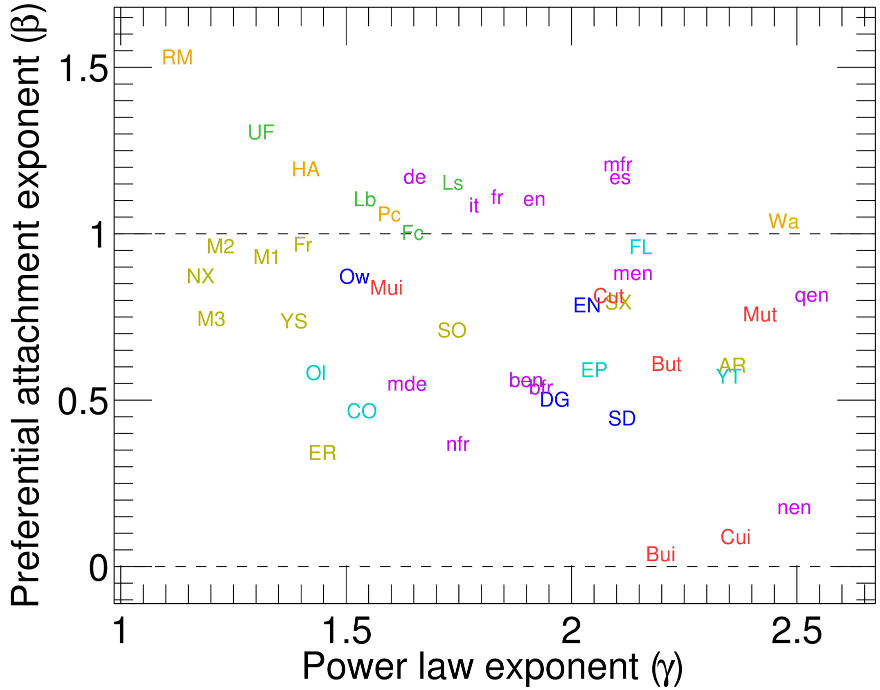




Facebook Wall Post Preferential Attachment



Comparison



■	Social network	$\beta < 1$
■	Rating network	$\beta < 1$
■	Communication network	$\beta < 1$
■	Folksonomy	$\beta < 1$
■	Wiki edit network	
■	Explicit interaction network	$\beta > 1$
■	Implicit interaction network	$\beta > 1$